STUDY OF VELOCITY AND ACCELERATION

AIM: To determine the value of g and velocity of an object rolling down an incline and compare them with the theoretical values. Learn the Excel and it's plotting function.

APPARATUS:

1. Three cylindrical shaped objects made up of plastic, wood, and steel with same dimension but different known mass (diameter: 2 cm, height: 15 cm)
2. Wooden plank (length: minimum 98 cm, width: 30 cm) attached with a rubber sheet on one side of the plane.
3. Meter scale (steel, length :30 cm)
4. Mobile Phone Stop Watch.
5. A steel stand of height 50 cm with the markings for every centimeter and a provision to hold the wooden plank in different angles from the ground.

THEORY:

The following concepts are explaining based on the solid cylinder moving through an inclined plane shown below.

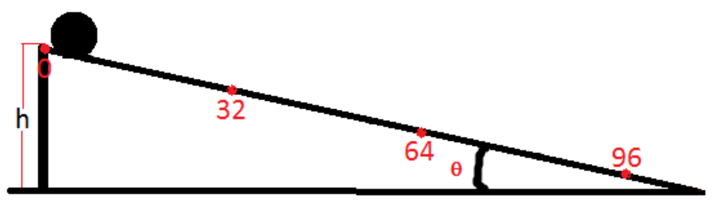


Fig 1: Arrangement of inclined plane

Moment of inertia (I): A quantity expressing a body's tendency to resist angular acceleration, which is the sum of the products of the mass (M) of each particle in the body with the square of its distance(R) from the axis of rotation. The moment of inertia of a solid cylinder can be expressed as

$$I= \frac{1}{2} MR^2$$

Angular momentumcan be defined as

$$I\omega=I(v/R)=(I/R)v$$

Torque: Total torque on the cylinder when it rolls without slipping is the rate of change of its angular momentum.

$$d(I\omega)/dt=d((I/R)v) / dt= (I/R) a$$

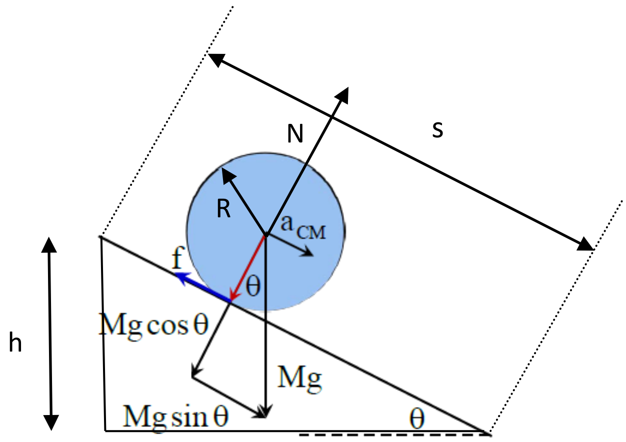
Frictional Force: Frictional force on the cylinder during rolling is $\mu Mg\cos\theta$ (μ is the coefficient of friction). The frictional force is normal to the radius of the cylinder's axis of rotation (when it rolls) and it is applied at the surface of the cylinder (a distance R from the axis), so friction contributes a torque on the ball equal to $R.\mu.M.g\cos\theta$

In order for the ball not to slip, the torque on the ball due to friction cannot be less thanthe total torque on the ball when it rolls, and therefore $RMg\mu_0\cos\theta \geq (I/R)a$ or

$$\mu_0 \geq k.\, a/(g\cos\theta) \quad \text{with } k = I/MR^2$$

Motion of cylinder rolling down an incline:

Translation motion of a body is obtained by assuming that all the external forces act at its centre of mass. Using Newton's second law, F=ma



The rotational motion (Torque) about the centre of mass follows

$$\tau = Rf = I\alpha$$

where I is Moment of Inertia and α is rotational acceleration.

The frictional force on the object is given by

$$f = \frac{I}{R}\alpha$$

where R is the radius of the cylinder.

Net force acting on the cylinder which causes the linear motion down the incline,

$$Mg\sin\theta - f = Ma_{CM} \qquad (1)$$

$$Mg\sin\theta - \frac{I}{R}\alpha = Ma_{CM} \qquad (2)$$

Where a_{CM} is acceleration down the incline, g is the acceleration due to gravity.

θ is the angle of inclination of the ramp.

$\theta = \sin^{-1}(h/s)$

The linear acceleration of the object is

$a_{CM} = R\alpha$,

$$\alpha = \frac{a_{CM}}{R}$$

From equation (2), we can write,

$$Mg\sin\theta - \frac{I}{R}\alpha = Ma_{CM}$$

$$Mg\sin\theta - \frac{I}{R}\frac{a_{CM}}{R} = Ma_{CM} \quad \rightarrow \quad -Ma_{CM} - \frac{I}{R^2}a_{CM} = -Mg\sin\theta$$

Multiplying throughout by -1,

$$Ma_{CM} + \frac{I}{R^2}a_{CM} = Mg\sin\theta$$

$$Ma_{CM}\left(1 + \frac{I}{MR^2}\right) = Mg\sin\theta$$

$$a_{CM}\left(1 + \frac{I}{MR^2}\right) = g\sin\theta$$

So that, finally we get,

$$a_{CM} = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

The object is a solid cylinder. So,

$$I = \frac{1}{2}MR^2$$

Where I is the moment of inertia of the cylinder.

$$a_{CM} = \frac{g\sin\theta}{1 + \frac{\frac{1}{2}MR^2}{MR^2}} = \frac{g\sin\theta}{1 + \frac{1}{2}} = \frac{g\sin\theta}{\frac{3}{2}} = \frac{2}{3}g\sin\theta \quad \text{(solid cylinder)}$$

Velocity of the object is

$v^2 = u^2 + 2as$

where u is initial velocity, v is final velocity and s is the displacement.

$v^2 = 2as$

$v^2 = 2*(2/3)*g*\sin\theta*s$

$v^2 = 2*(2/3)*g*(h/s)*s$

$v = \sqrt{\frac{4\,g\,h}{3}}$

PROCEDURE :

Step 1: On the wooden plank,draw lines in the intervals of 32cm (say).

Step 2: Now incline the wooden plank at a height of 5cm (say).

Step 3: Find the angle θ.

Step 4: Place a cylinder (say aluminium) on top of the ramp.

Step 5: The moment you leave the object, the stopwatch is turned on simultaneously.

Step 6: As the object rolls down the incline, measure the time taken to cross each marked lines at distances of 32 cm (i.e.,)

Measure t1 at s1 = 32 cm.

Measure t2 at s2 = 64 cm.

Measure t3 at s3 = 96 cm.

Step 7: Repeat step 4 to step 6 for 30 trials

Step 8: Repeat step 4 to step 7 for remaining two cylinders

The following table can be used to record the values of three different objects.

Object : Plastic		**Mass (M) :**				
Trails	**s1**	**t1**	**s2**	**t2**	**s3**	**t3**
1						
2	.32		.64		.96	
30						
Average						
Object : SS		**Mass (M) :**				
Trails	**s1**	**t1**	**s2**	**t2**	**s3**	**t3**
1						
2	.32		.64		.96	
30						
Average						
Object : Aluminium		**Mass (M) :**				
Trails	**s1**	**t1**	**s2**	**t2**	**s3**	**t3**
1						
2	.32		.64		.96	
30						
Average						

s1, s2, s3 are the distances in meters.

t1, t2, t3 are the time taken to cover each distance.

Step 9: Find acceleration and velocity of each rolling object.

$s = ut + 0.5at^2$

u=0 (Initial velocity of the rolling object =0)

$s_1 = 0.5at_1^2$

$s_2 = 0.5at_2^2$

$s_3 = 0.5at_3^2$

$s_2 - s_1 = 0.5*a*(t_2^2 - t_1^2)$

$s_3 - s_1 = 0.5*a*(t_3^2 - t_1^2)$

$s_3 - s_2 = 0.5*a*(t_3^2 - t_2^2)$

Step 11:

X	t_1^2	t_2^2	t_3^2	t_2^2 - t_1^2	t_3^2 - t_1^2	t_3^2 - t_2^2
Y	s_1	s_2	s_3	$s_2 - s_1$	$s_3 - s_1$	$s_3 - s_2$

Step 12: Plot Y Vs X and find the slope of the trend line from the excel sheet.

Slope = 0.5a

Step 13: After finding 'a' from step 12, find v (velocity) by

$v = u + at$ => $v = at$

$v_1 = at_1$

$v_2 = at_2$

$v_3 = at_3$

Step 14: Plot v Vs t.

Inference: